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CONTINUOUS ESTIMATION OF LONGSHORE SAND TRANSPORT

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A. Definition of the Problem

Effective resource management for sandy coastlines requires the budgeting of sediment. This activity can be passive, as expressed in planning functions, or active, when the natural sediment distributions are manipulated for man's benefit. In either case, success is dependent upon an adequate knowledge of the gross -- or in certain cases the net -- longshore transport of sediment by waves. Some examples will illustrate the application of this knowledge to coastal resource management.

Long-term weather cycles and upland flood control measures have denied sediments to a source delta. As a result, the downdrift coastline is suffering erosion. The acceptable alternatives are artificial beach renourishment or abandonment of ocean front improvements. To evaluate the cost of the renourishment project, it is necessary to know the net transport out of the affected areas in order to know how much sand must be put on the beach and how often the treatment might need repeating.

A harbor is considered for the sandy coastline. The maintenance dredging costs are directly related to the gross transport past the harbor entrance. One or both of the fillets adjacent to the entrance jetties, depending upon the transport characteristics, may need artificial filling initially to supply downdrift needs.

An existing harbor has a serious entrance shoaling problem, and conventional dredging is extremely costly because of high waves outside the entrance. A permanent sand bypassing system is planned. The capacity of the bypassing plant is related to the gross transport. In addition, the requirement to bypass in either one or both directions can only be determined from an understanding of the seasonal variations in the transport directions.

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In each of these examples, the anticipated construction cost for moving the sand is roughly proportional to the natural net transport. In other words, cost projections can be no better than the transport projections. The economic leverage is obvious.

Accurate estimation of net transport rates, however, is extremely difficult. The presently employed models for longshore transport, described in the Shore Protection Manual (SPM 1973), relate the gross transport empirically to the breaker energy and approach angle. The scatter in the observed data suggests that an estimate of gross transport using this method might easily be in error by a factor of three or four. If the longshore transport is bi-directional -- as it is along much of the coastline of the United States -- the net transport is the small difference between two large numbers. Therefore, the error in estimating the gross transports can easily be an order of magnitude larger than the net transport; and, it is possible that even the predicted direction of the net transport will be wrong.

A second problem exists with the implementation of the present transport models in the requirement for measurement or estimation of breaker height and breaker approach angle. Since the waves break over a broad zone determined by the variation in wave height and the tidal range, no fixed instruments can measure these parameters. Therefore, these measurements, with present technology, can only be made by individuals willing to venture into the surf and are thus limited to relatively small waves. Visual observations of these parameters from shore, of questionable accuracy under any conditions, become hopeless with the broad surf zones associated with storm conditions -- the very conditions that are most important to observe.

Lacking the capability, with existing technology, to make continuous measurement of sediment in motion past a point, the actual transport must be inferred from some measurement of the forcing function -- the waves. For the estimate to account for the extreme variability of this forcing function, the wave observations should be made several times per day. For such a continuous observation program to be affordable implies that it is automatic and requires no on-site personnel. Since the only available transport models are based upon parameters that are unmeasurable by an automatic system, this further implies that some means must be found of estimating breaker angle and breaker height from some other measurable characteristics of the waves.

B. Estimation Theory

The longshore transport models can be written in the form

$$Q = K(\frac{E}{2} C_g \sin 2\theta)_b \quad (1)$$

where Q = mass transport
K = empirical constant defining transport
E = wave energy density
C_g = group velocity
θ = approach angle

The subscript, $_b$, indicates that all of the quantities within the bracket are evaluated at the breaker location.

When a wave train in deep water approaches the shore at an oblique angle, there is a longshore component of the momentum transported in the direction of wave advance. Longuet-Higgins (1970) shows that for coastlines with parallel contours, S_{xy}--the longshore component of momentum flux -- remains constant from deep water into the breaker zone where it is reacted by bottom friction and drives a longshore current within the surf zone. It is this current which carries downcoast the sediment dislodged by wave agitation.

Longuet-Higgins (1970) provides the following formulation for this momentum flux component

$$S_{xy} = \frac{1}{2} \sum_f E \frac{C_g}{C} \sin 2\theta \quad (2)$$

where $\sum_f$ = sum over all frequencies
and C = phase velocity

Evaluating equation (2) at the breaker point yields

$$(S_{xy})_b = (\frac{E}{2} \sin 2\theta)_b \quad (3)$$

where the summation is applied only to the energy since all frequency components now have the same approach angle, θ, and where the ratio C_g/C is assumed to be unity.

A comparison of Equation (3) with the transport Equation (1) shows immediately that

$$Q = K(C_g S_{xy})_b \quad (4)$$

However, since S_{xy} is constant seaward of the breaker point under straight coastline conditions, Equation (4) can be rewritten eliminating the $_b$ subscript on S_{xy}.

$$Q = K(C_g)_b\, S_{xy} \tag{5}$$

The group velocity in shallow water is given by

$$(C_g)_b = (gh_b)^{1/2} \tag{6}$$

where h_b = depth at the breaker point

g = gravitational constant

The SPM contains empirical graphs relating breaker height with offshore wave height using the beach slope and the offshore steepness. An estimate of the offshore significant wave height can be made from the variance of the surface elevation using the relationship

$$H_s = 4\sigma_n \tag{7}$$

where H_s is the significant wave height at the measurement point outside the surf zone

and σ_n is the measured standard deviation of the surface elevation at that point.

This value of significant wave height and the appropriate deep water wave length for the period of peak energy in the measured wave spectrum can be used to construct a deep water steepness ratio to employ in the SPM graph. The significant breaker height can then be used in a second SPM graph, using beach slope, to estimate the depth at breaking, h_b.

It can be seen from Equations (5) and (6), however, that the transport estimate has a relatively low sensitivity to small errors in estimating h_b. Therefore, a simpler alternate method is available. An estimate of the breaker height is obtained by a linear empirical predictor from the measured significant wave height. The depth at breaking is then assumed to be equal to the breaker height. Griswold (1964) gives a formulation of the form

$$h_b = 1.65\ H_s \tag{8}$$

Combining Equations (5), (6) and (8) gives

$$Q = K\ S_{xy}\ (1.65\ g\ H_s)^{1/2} \tag{9}$$

Therefore, the longshore transport rate can be estimated knowing only the significant wave height and the longshore component of momentum flux measured outside the surf zone -- provided that the measurement location satisfies the requirement for straight and parallel bottom contours between the measurement point and the surf zone.

C. Estimating S_{xy}

Longuet-Higgins, et. al. (1963), shows that S_{xy} can be estimated from the sea surface slope components by

$$S_{xy} = \frac{\rho g}{2} \sum_f \frac{n}{K^2}\ C_{x,y} \tag{10}$$

where ρ = fluid density

$n = \frac{C_g}{C}$, the ratio of group and phase speeds

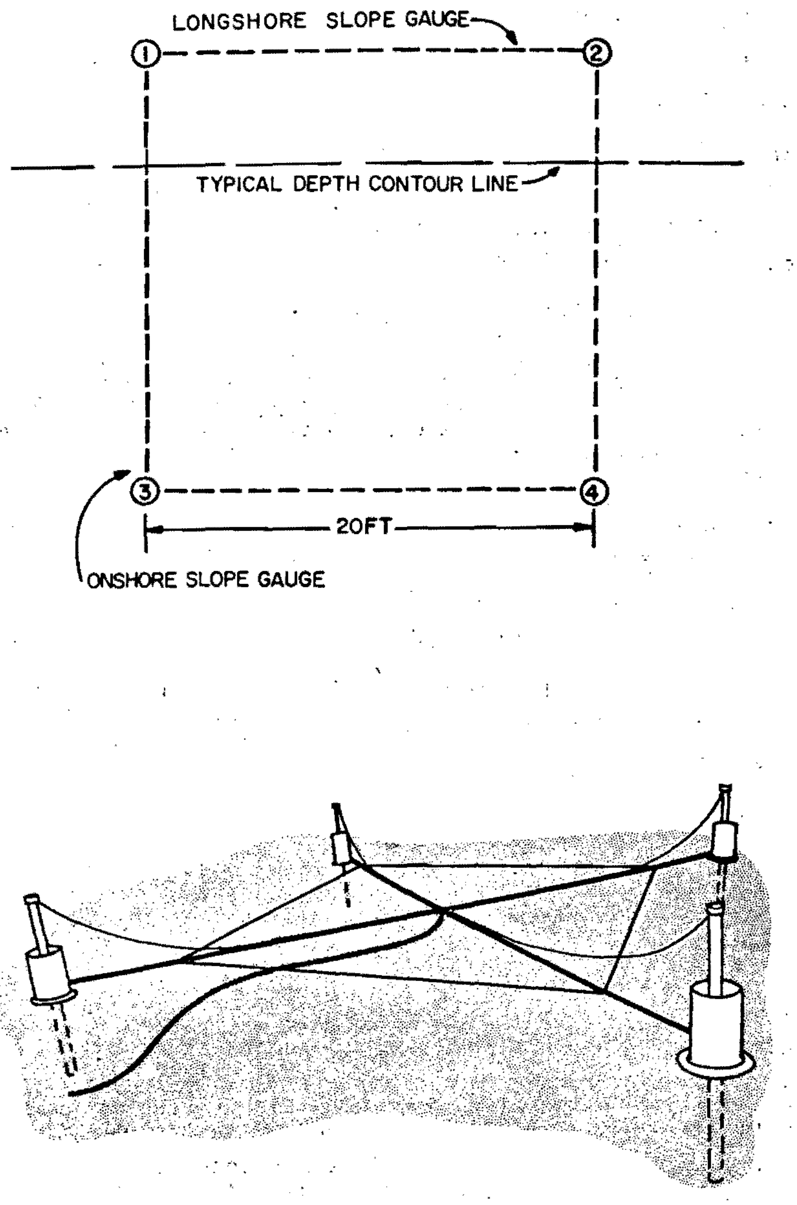
K = wave number

and

$C_{x,y}$ = real part of the cross-spectrum of the sea surface slope components in the onshore and longshore directions.

An economical method for measuring the slope components of the sea surface is described by Seymour and Higgins (1977). The instrument is known as a slope array and consists of four pressure transducers mounted on a compact square frame, six meters on a side, as shown in Figure 1. The plan view of the frame shows an idealized installation with the longshore slope gage pairs aligned parallel to the trend of local contours of depth -- the reference direction for S_{xy}. In practice, this is not required since a simple coordinate rotation yields the desired slope components from any array orientation. The signals from the pressure transducers are carried ashore through armored cables.

The slope spectra can be approximated by transforming the time series of the differences between surface elevations in a pair, divided by the horizontal distance between gages.



SLOPE ARRAY FOR MEASURING WAVE DIRECTION
FIGURE 1

Since the pressure record is not a linear estimator of the surface elevation in the time domain, it is more convenient in this instance to find the difference between the transforms of the pressure records, divide these values by the gage separation, and use linear theory to correct the resulting transform to a surface elevation slope transform. The transforms of onshore and longshore slopes are then used to calculate the co-spectrum terms in Equation (10). The values of n and K in this equation are calculated through linear theory for each frequency interval. The sum overall frequency interval provides an estimate of S_{xy}, the longshore component of wave-generated momentum flux.

D. Estimating Longshore Transport

The value for the empirical coefficient, K, in Equation (9) has been obtained by fitting to a limited number of field data sets -- all with known deficiencies in either the definition of the wave field or in the measurement of the sediment response. However, the presently accepted value as shown in SPM represents the best available estimator.

Expressing S_{xy} in dyne/cm and C_g in cm/sec, the SPM value can be converted to

$$K = 0.0129 \frac{m^3}{yr} \frac{sec}{dyne} \quad (11)$$

If S_{xy} is represented in the more customary units of cm^2, by dropping the $\frac{\rho g}{2}$ term, and H_s is measured in cm, then Equation (9) can be rewritten as

$$Q(m^3/yr) = 255\ S_{xy}(cm^2)\ (H_s(cm)^{1/2}) \quad (12)$$

Although this method does not require the calculation of wave approach angles, it is intuitively satisfying to assign some direction to the wave fields that cause the transport.

Inspection of Equation (2) shows that knowing S_{xy}, a single angle, $\hat{\theta}$, can be assigned that will satisfy the equation. Although each frequency component may have its own unique approach angle, $\hat{\theta}$ represents a measure of the wave field directionality in that it is the one approach angle which would result in the same S_{xy} as is calculated from summing all of the directional components. $\hat{\theta}$ is referred to, hereafter, as the significant approach angle. Although S_{xy} is everywhere constant outside the surf zone, $\hat{\theta}$ is not, so that its location must be specified.

E. Application to Santa Cruz, California

The U. S. Army Engineers, concerned with entrance siltation problems at Santa Cruz Harbor, has funded the Scripps Institution of Oceanography to install a slope array at this location to allow estimation of longshore sediment transport. Correlation between these estimates and observed sediment deposition rates -- as determined from dredging and bypassing records -- will be attempted by the Army Engineers. Data from the measuring system at Santa Cruz are collected and analyzed through the California Coastal Engineering Data Network which is operated by the Department of Navigation and Ocean Development. This system is described in Seymour and Sessions (1976). The array is mounted at a depth of 7 meters. The pressure transducers are sampled at 1 Hz for 1024 seconds approximately every 10 hours.

The Santa Cruz Harbor installation provides an example of the method for estimating longshore transport described in this paper. A thirty-day period between 26 September and 25 October 1977 was selected to illustrate the method.

Figure 2 shows the day-to-day variation in the average values of signifigant wave height, significant angle and S_{xy} during this period. Figure 3 shows the daily estimate of longshore transport and the cumulative (net) transport.

Consideration of these two figures provides the following observations:

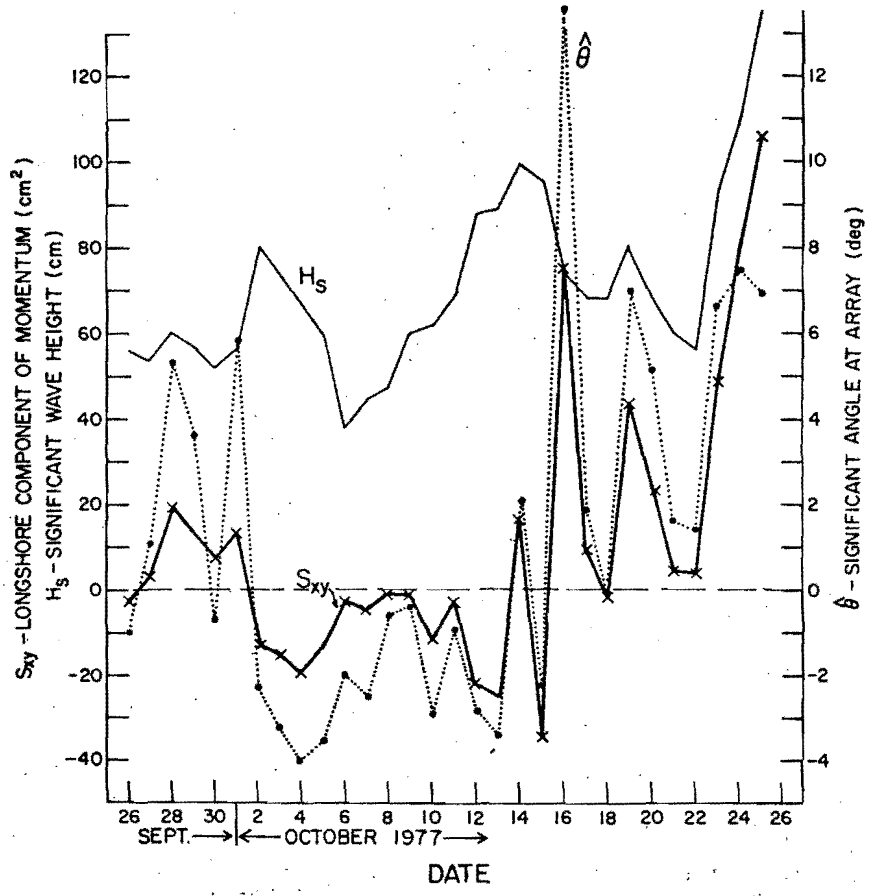
1. Significant angles are quite small -- seldom exceeding 6° in magnitude. Even under the conditions of large transport in the last few days, $\hat{\theta}$ is less than 8°. Breaker angles, because of continuing refraction, will be even smaller.

2. Even though H_s varies between 0.5 and 1.0 m during the first three weeks of the period, the net transport is nearly zero.

3. Significant net transport occurs only under conditions of very high waves during the last three days of the period.

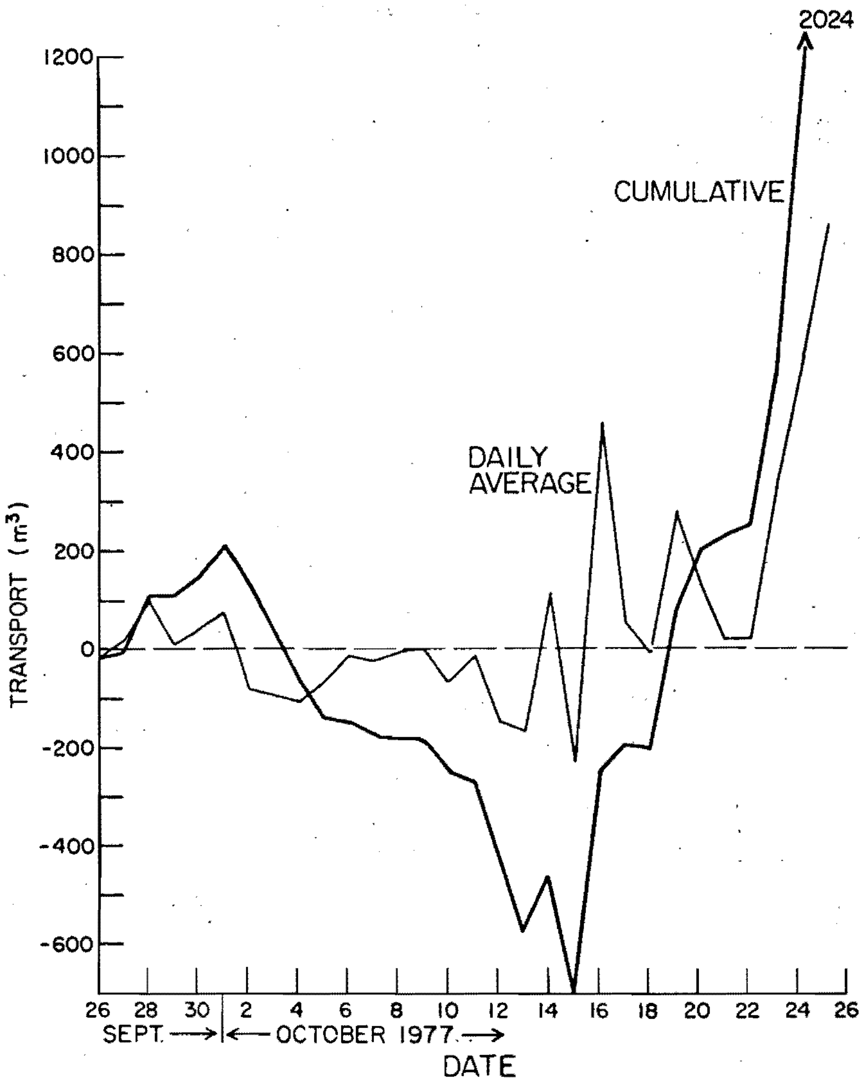
F. Observations and Conclusions

A simple, small and compact slope array of at least three pressure transducers appears to provide reasonable estimates of longshore transport. During the thirty-day period described, no detrimental accretion of sediment was observed in the harbor entrance adjacent to the slope array prior to the late October event, when substantial shoaling occurred. Thus, the gross features of the estimated



DAILY AVERAGE VALUES, SANTA CRUZ, CA.

FIGURE 2



LONGSHORE TRANSPORT ESTIMATES, SANTA CRUZ, CA.

FIGURE 3

transport agree with observations.

Even though peak transport rates of 450 M^3 per day (164,000 M^3 per year) were achieved in the first three-week interval, the net transport was nearly zero. This emphasized the problems associated with predicting the difference between two large gross transport components to arrive at net transport.

The small approach angles, even in 7 meters of depth, suggest that hindcasting waves in deep water and applying refraction theory may not achieve sufficient accuracy to predict net transport.

REFERENCES

1. Griswold, G. M. Surf Forecasting. Navy Weather Research Facility, Norfolk, Virginia, Report No. 36-1264-099, December, 1964.

2. Longuet-Higgins, M. S., Cartwright, D. E., and Smith, N. D. "Observations of the Directional Spectrum of Sea Waves Using the Motions of a Floating Buoy," Ocean Waves Spectra, Proceedings of a Conference, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963.

3. Longuet-Higgins, M. S. "Longshore Currents Generated by Obliquely Incident Sea Waves," Journal of Geophysical Res., Vol. 75, No. 33, November 20, 1970, pp. 6778-6789.

4. Seymour, R. J. and Sessions, M. H. "A Regional Network for Coastal Engineering Data," Proceedings of the Fifteenth International Conference on Coastal Engineering, Honolulu, Hawaii, July, 1976.

5. Seymour, R. J. and Higgins, A. L. "A Slope Array for Estimating Wave Direction," Proceedings of a Workshop on Instrumentation for Nearshore Processes, Institute of Marine Resources, University of California at San Diego, June, 1977. (in press)

6. Shore Protection Manual, Vol. 1, Coastal Engineering Research Center, U. S. Army Engineers, Washington, D. C., 1973.