A Method for Identifying Kolmogorov’s Inertial Subrange in the Velocity Variance Spectrum

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ABSTRACT

Kolmogorov’s inertial subrange is one of the most recognized concepts in fluid turbulence. However, the practical application of this theory to turbulent flows requires identifying subrange bandwidth. In the atmospheric boundary layer, decades of investigation support Kolmogorov’s theory, but the techniques used to identify the subrange vary and no systematic approach has emerged. The algorithm for robust identification of the inertial subrange (ARIIS) has been developed to facilitate empirical studies of the turbulence cascade. ARIIS systematically and robustly identifies the most probable subrange bandwidth in a given velocity variance spectrum. The algorithm is a novel approach in that it directly uses the expected 3/4 ratio between streamwise and transverse velocity components to locate the onset and extent of the inertial subrange within a single energy density spectrum. Furthermore, ARIIS does not assume a $-5/3$ power law, but instead uses a robust, iterative statistical fitting technique to derive the slope over the identified range. This algorithm was tested using a comprehensive micrometeorological dataset obtained from FLIP. The analysis revealed substantial variation in the inertial subrange bandwidth and spectral slope, which may be driven, in part, by mechanical wind-wave interactions. Although demonstrated using marine atmospheric data, ARIIS is a general approach that can be used to study the energy cascade in other turbulent flows.
1. Introduction

Nearly eight decades ago, Kolmogorov proposed a set of theories that described a universal structure of small-scale turbulent motion within a homogeneous fluid under high Reynolds ($Re$) numbers (Batchelor 1947, 1953). These theories have been applied to the experimental study of fluid turbulence across a wide range of flows from the laboratory to geophysical scale. The atmospheric surface layer (ASL) is considered to be at sufficient $Re$ to develop the turbulence structure Kolmogorov identified. Following the original publication (Kolmogorov 1941b,a) various studies corroborated his theories, thereby establishing a general acceptance of their validity within the atmosphere (MacCready 1962; Miyake et al. 1970; Atta and Chen 1970; Kaimal et al. 1972).

One of the impacts of Kolmogorov’s theories on the study of the ASL, is the use of the inertial dissipation method (IDM) to estimate the momentum flux, as well as the rate of turbulence kinetic energy ($TKE$) dissipation ($\varepsilon$) near the surface (e.g. Large and Pond 1981, 1982; Fairall and Larsen 1986). Based on the assumptions Kolmogorov made to develop his theories, dimensional analysis yields the TKE spectral density over the inertial subrange,

$$E(k) = \alpha \varepsilon^{2/3} k^{-5/3},$$  \hspace{1cm} (1)

which has become one of the most widely recognized equations in the study of geophysical fluid turbulence. $\alpha$ is an universal proportionality constant (Yaglom 1981; Deacon 1988; Hogstrom 1996; Yeung and Zhou 1997) and $k$ is wavenumber. The spectrum represents the inviscid cascade mechanism (Onsager 1945) from the energy containing eddies to the dissipation spectrum (Tennekes and Lumley 1972). Assuming one can...
measure $E(k)$, an estimate of $\varepsilon$ may be derived, which in turn can be related to the friction (or shear) velocity, $u_\ast$. Following Yelland and Taylor (1996):

$$\varepsilon = \frac{u_\ast^3}{\kappa z} \phi_\varepsilon, \tag{2}$$

where $\kappa$ is the Von Kármán constant, $z$ is the transverse height into the surface layer, and $\phi_\varepsilon$ is the non-dimensional dissipation function from Monin-Obukhov Similarity Theory (MOST). $\phi_\varepsilon$ is a function of stability, $\zeta = z/L$ ($L$ is the Obukhov length), and must be determined empirically (e.g. Large and Pond 1981; Yelland and Taylor 1996). A corollary can be made in the ocean where the same formulae are applied to investigating developed turbulence in stable boundary layers and the dependence on the Dougherty-Ozmidov length scale (Grachev et al. 2015).

Operationally, IDM is advantageous because it is usually evaluated over a subrange of $k$ where the influence of platform motion, flow distortion, and surface gravity waves (in the case of the marine environment) are negligible. However, IDM remains an indirect method and the advent of higher quality and cost-effective motion systems has promoted the eddy covariance technique for deriving flux estimates (Edson et al. 2013). In spite of this, IDM persists as a viable method with continued application in both field and numerical studies (Jabbari et al. 2015; Hackerott et al. 2017).

One practical challenge to using IDM is determining the appropriate subrange of $k$. Kolmogorov did not provide explicitly for identifying the appropriate subrange and, while (1) is expected to be widely valid, the bandwidth of $k$ ($\Delta k$) over which it applies depends on $Re$ (Batchelor 1953; Tennekes and Lumley 1972). In reviewing the ASL literature, a sample of prominent and recent studies revealed no consistent method or ap-
approach for identifying $\Delta k$ in observational data (Payne and Lumley 1966; Miyake et al. 1970; Kaimal et al. 1972; Kaimal 1978; Large and Pond 1981, 1982; Fairall and Larsen 1986; Edson et al. 1991; Durand et al. 1991; Anderson 1993; Yelland and Taylor 1996; Edson and Fairall 1998; Sjöblom and Smedman 2002; Hackerott et al. 2017; Muñoz-Esparza et al. 2018). In fact, only three of those listed here directly state what bandwidth was used and how this was determined. Yelland and Taylor (1996) and Muñoz-Esparza et al. (2018) both used fixed bandwidths for the inertial subrange from 2-4 Hz and 0.5-10 Hz, respectively, with no reported justification. Sjöblom and Smedman (2002) reported that they visually inspected their spectra and rejected those that did not appear to follow a $-5/3$ slope. As an oceanic turbulence example, Grachev et al. (2015) derived estimates for $\varepsilon$ using IDM over a fixed bandwidth [0.49,0.74] Hz, which they determined to be within the inertial subrange. Across studies over land and ocean, the most common approach to identifying the inertial subrange, or validating its presence, was visually comparing the measurement-derived power spectrum to a line with log-scaled slope of $-5/3$. Additionally, it was rare for investigators to give any indication of the prevalence of an inertial subrange in their data or any sense of the relative uncertainty. While this may not have been critical for results in all the studies listed, it highlights an approach across decades of study that is highly idiosyncratic and reflects some of the most highly cited studies on the subject (aggregate Google Scholar citations $>$7500).

Recently, Hackerott et al. (2016) presented an iterative, inertial subrange detection method (see Appendix of that paper). The underlying basis of the method was to find the $\Delta k$ where $\alpha$ converges, to within $\pm 10\%$ of one standard deviation. This algorithm
is more robust than visual inspection and indirectly relies on Kolmogorov’s underlying
assumption of isotropy (discussed further below). However, the Hackerott et al. (2016)
method relies on an estimate of $\varepsilon$ (via polynomial fitting of the log-linear portion of the
spectrum) and the universality of the $-5/3$ power law. A similar methodology was em-
ployed by Jabbari et al. (2015) to study simulations of open channel flow. While meth-
ods like this exist in the physics literature, in the context of the ASL, techniques appear
to be ad hoc and the documentation of what specifically was done is not consistently
reported. Furthermore, visual inspection persists as a credible means for identifying
Kolmogorov’s inertial subrange within a given spectrum.

Here, it is argued that the lack of a consistent or efficient technique for detecting
the inertial subrange in observed or modeled turbulence spectra presents a significant
oversight in the meteorological community. In order to address this, the algorithm for
robust identification of the inertial subrange (ARIIS) was developed. While the idea of
devising a technique to locate the inertial subrange is not novel, having been necessary
since the theory’s 1941 proposal, there is a need for a standard approach that can be
easily incorporated into any investigator’s analysis toolbox. This is akin to the way
certain eddy covariance flux algorithms have become conventional and been widely
utilized in studying a diversity of ASL data sets (e.g. COARE). The primary objective
of ARIIS is to move away from an over-reliance on the eyeball test as a valid assessment
for the presence and location of Kolmogorov’s inertial subrange. The scope of this
article will be to detail the basis and implementation of ARIIS, using a field data set
collected from the FLoating Instrument Platform (FLIP) as an example. Some analysis
of the results from FLIP will also be discussed, but the in-depth analyses exploring the
results of this particular data set has been summarized in a separate article by the authors
that is currently in preparation for peer-reviewed publication.

2. Kolmogorov’s Inertial Subrange

The basis for Equation (1) will be outlined below and, unless otherwise noted, the
source of this review comes from Batchelor (1953) and Tennekes and Lumley (1972).
The former is based largely on one of the first English translations of the original articles
published in Russian (see Batchelor 1947). This work will only focus on Kolmogorov’s
formulae as they relate to the velocity variance spectrum.

Given a homogeneous and incompressible fluid at sufficiently high $Re$, a portion of the
total turbulent energy spectrum can be considered in equilibrium with local conditions
and is thus independent of the mean. If all of the inertial input at low $k$ is dissipated by
viscosity ($\nu$) at high $k$, then the energy across this equilibrium spectrum is functionally
defined as:

$$E = E(k, \varepsilon, \nu).$$

Since Equation (3) is independent of the mean variance, this relationship should be valid
for any flow and is known as Universal Equilibrium Theory.

Dimensional analysis of $E$ yields one possible result,

$$E = \nu^{5/4} \varepsilon^{1/4} f(k\eta),$$

where the Kolmogorov microscale, $\eta = (\nu^3 / \varepsilon)^{1/4}$, is the upper limit of $k$ in (4). The
eddies across $E$ are also characterized by the Kolmogorov velocity scale, $v = (\nu \varepsilon)^{1/4}$,
so that (4) can be written as,

\[ E = \nu^2 \eta f(k \eta). \]  

Therefore, to predict \( E(k) \), one must determine the form of \( f \), which may be complicated. However, Kolmogorov argued that as \( Re \) continued to increase (beyond the cut-off for Equation 3 to be valid) a subrange of \( E \) will develop where both influence of the mean and \( \nu \) could be neglected. This inviscid portion of \( E \), would then be solely controlled by \( \epsilon \) and thus considered an inertial subrange. Assuming a power law in (4),

\[ E \sim \nu^{5/4} \epsilon^{1/4} f(k \nu^{3/4} \epsilon^{-1/4}) \]

it can be seen that to eliminate viscosity from \( E \), \( \delta \) must have the value,

\[ 1 = \nu^{5/4} \nu^{3 \delta/4} \]

\[ \nu^{-5/4} = \nu^{3 \delta/4} \]

\[ -5/4 = 3 \delta/4 \]

\[ \delta = -5/3. \]  

Substituting this \( \delta \) into (6), we arrive again at Equation 1,

\[ E(k) = \alpha \epsilon^{2/3} k^{-5/3}, \]

which will be referred to as Kolmogorov’s spectrum or the inertial subrange spectrum. The \( k \)-dependence of this spectrum will be referred to as either the power law or inertial subrange slope. This relation is only valid over a specific subrange, or bandwidth \( \Delta k \), where \( E(\Delta k) \) satisfies these conditions:

1. \( E(\Delta k) \) is isotropic over \( \Delta k \),
2. \( E(\Delta k) \) exhibits a \(-5/3\) dependence on \( k \).

Both of these conditions must be simultaneously true for Kolmogorov’s spectrum to be valid (Yeung and Zhou 1997). To emphasize, a subrange within any spectrum exhibiting \( k^{-5/3} \) does not necessarily signify that that portion is the inertial subrange, or even the presence of the inertial subrange anywhere within the total spectral bandwidth. The condition of isotropy must be satisfied because of Kolmogorov’s original argument that the turbulent motions over this subrange are independent from the inherently anisotropic mean inertial motions (Tennekes and Lumley 1972). The second condition may appear circular, but it underscores that the \(-5/3\) power law is the only possible, dimensionally consistent outcome for Equation 4, assuming an inviscid cascade from low to high \( k \). Therefore, condition #1 may be considered the more fundamental of the two, since it is based on the truly fluid mechanical idea of a homogeneous, high \( Re \) flow in equilibrium.

\( E \) is the total TKE spectrum, which can be related to the one-dimensional velocity variance spectrum, \( S_{\beta\beta}(k) = a_\beta E(k) \). For the purposes of this paper, \( \beta \) can be either the stream-wise (\( u \)), horizontal transverse (\( v \)), or vertical (\( w \)) velocity components. Over the inertial subrange, there is a predictable relationship between \( S_{uu} \) and \( S_{vv,ww} \),

\[
S_{uu} = \frac{3}{4} S_{vv,ww},
\]

and so, an isotropy coefficient, \( I \), can be defined (Jimenez et al. 1992),

\[
I(k) = \frac{S_{uu} - k \partial S_{uu}/\partial k}{2S_{vv,ww}},
\]

such that \( I(\Delta k) \to 1 \).
Typical ASL turbulence studies derive $S_{ββ}$ from high-temporal resolution anemometry. Therefore, turbulence is not resolved in terms of $k$, but rather frequency, $n$. The transformation from spatial to temporal scale requires invoking Taylor’s Frozen Turbulence theory, which allows $k = 2\pi n/U$, where $U$ is the mean advection velocity. Utilizing $n$ versus $k$ does not result in any fundamental change to the Kolmogorov spectrum or its implications and so the inertial subrange frequency bandwidth can be defined as $Δn$.

It should be noted that there is some controversy with how to properly apply Taylor’s theory (e.g. Wyngaard and Clifford 1977) to account for a frequency-independent offset (Edson and Fairall (1998) report a $\sim 2\%$ change in spectral amplitude), but this is not as critical here since the primary focus is for the slope of $S_{ββ}(n)$, which should not be effected by this correction.

In the ASL, some anisotropy is expected (Wyngaard 2010) and has been observed using sonic anemometry for the case of thermodynamically stable and heterogeneous conditions (Babić and Rotach 2018). In this previous work, it was found that in fact $S_{uu}/S_{vv} \neq S_{uu}/S_{ww}$, with the left-hand side being $> 3/4$. This was observed using three-dimension sonic anemometers (Gill WindMaster Pro) and created uncertainty in their estimates of $ε$. While this previous work was in a unique regime (i.e., stable conditions, canopy fetch), these interesting findings suggest that examining $I(n)$ in all dimensions may be important to characterizing the high-frequency turbulence.

[ Figure 1 approximately here ]
3. Field Dataset

The data used to develop and test ARIIS was collected as part of the Coupled Air Sea Processes and Electromagnetic ducting Research (CASPER) west coast field campaign that took place south of the Channel Islands, offshore of Southern California in September and October of 2017 (Figure 1). CASPER focused on understanding how variability within the marine atmospheric boundary layer (MABL) impact the propagation of electromagnetic and electro-optical radiation over the ocean (Wang et al. 2018). CASPER-West was an intensive air-sea interaction study where coordinated measurements were made from a variety of research vessels and platforms. This included the Research Vessel Sally Ride, FLIP, a suite of autonomous ocean gliders, and shore-based measurement systems as well as the network of CDIP/NDBC oceanographic/meteorological surface buoys that are stationed in the Southern California bight region. The present study will focus on the atmospheric measurements made from a mast installed on the port boom of FLIP (Figure 1).

FLIP is a 108 m long spar buoy and platform that uses water-filled ballast tanks to stay vertical at the ocean surface (Fisher and Spiess 1963). For CASPER-West, FLIP was tri-moored, which resulted in a natural vertical and azimuthal oscillation with period 63.38 seconds, a sub-harmonic was also present at approximately 33 seconds. While these motions were persistent, their absolute deviations were ~1-10 cm, which is significantly less than the motion experienced on a typical research vessel even under mild or moderate conditions. For this reason, FLIP remains one of the best platforms for
making undisturbed oceanographic and atmospheric measurements near the interface in a wide variety of environmental conditions.

*FLIP* is outfitted with booms that enable measurements to be made far away from platform’s superstructure. The port-side boom is approximately 18 m long and extends perpendicular from the main body (see Figure 1). At the end of the boom a 13 m long meteorological mast, aluminum triangular lattice, was emplaced and installed with a total of 17 atmospheric measurement levels. Across these levels, was arranged overlapping bulk and turbulence-resolving profiles, which included: combined 17 two- or three-dimensional (3D) sonic anemometers, 20 relative humidity probes, and 25 bulk temperature probes. The measurement capability of this mast represents an extensive characterization of the marine ASL (MASL) during CASPER-West and one of the most complete, near-surface atmospheric profile data sets available over the ocean. The port boom was also outfitted with atmospheric pressure sensors, an inertial motion unit (IMU) paired with each three-dimensional sonic anemometer, a laser altimeter (for instantaneous wave height under the boom), and a differential GPS (DGPS) antenna array for location and non-magnetometer-derived heading.

[ Figure 2 approximately here ]

This study presents analysis of the turbulence-resolving, or flux, measurements from multiple levels (i.e. profile) from the mast. The flux profile was comprised of seven 3D sonic anemometers, given in order of height above the ocean: 1 3D RM Young 81000 (3 m), a Campbell Scientific CSAT3/LiCOR LI7500 IRGA combo (4 m), and five Campbell Scientific IRGASON systems (5-16 m)–heights are approximate. The
exact altitude above sea level depended on various factors impacting *FLIP* displacement. The upper six levels resolved the momentum and total heat flux, the RM Young only resolves the momentum and sensible heat flux. The IRGASON systems were sampled at 50 Hz, while the other two systems were collected at 20 Hz. However, due to an internal low-pass filter the highest resolved frequency from the IRGASONs was 20 Hz. Co-located with each flux level (except the RM Young) was a VectorNav IMU (VN-100), that resolved the linear and rotational accelerations for each level of the mast at 50 Hz. The DGPS on the port boom was capable of precisely measuring the true heading of *FLIP* without using a magnetometer, which could be affected by the ferrous superstructure. The IMU and DGPS were blended at the high and low frequencies, respectively, and then used to motion-correct the raw anemometer velocities following (Edson et al. 1998). Before motion-correction, the raw velocity data was screened for spurious data using either internal diagnostics (for the IRGASONs) or a 20 sample-wide, moving median absolute deviation (MAD) filter. This approach is similar to some of the techniques described in the thorough, outlier-detection review given by Starkenburg et al. (2016) and which were determined to be the most reliable and stable. Any particular processing window where over 10% of the samples were flagged in the screening was excised from the final data set. Outlier samples in the preserved records were removed and the gap was interpolated using a linear polynomial. Further details on the instrument systems deployed and the data quality control and assessment of the CASPER-West measurements is provided in the report, Ortiz-Suslow et al. (2019), which also contains
256 a detailed review and analysis of the motion of FLIP during CASPER-West and how
257 this compares to previous FLIP studies.
258
259 Figure 2 highlights the general MASL conditions experienced during CASPER-West.
260 The focus here will be on the period September 30 to October 23 UTC. These conditions
261 were typical of the autumnal Southern California coastal region. Daily variability was
262 characterized by low, generally easterly wind in the local morning and transitioning to a
263 sea breeze in the local afternoon, $U \sim 5 \text{ ms}^{-1}$. The MASL was typically unstable with
264 a median ± m.a.d. $\zeta = -0.319 \pm 1.875$. The time record also experienced three Santa
265 Ana wind events, characterized by stratified conditions ($\zeta > 0$), and the passage of a
266 strong westerly front (Oct. 20-21) where sustained wind speeds exceeded 10 ms$^{-1}$ for
267 approximately 24 hours.

268

4. ARIIS

269 The primary aim in developing ARIIS was to efficiently and robustly identify the
270 most probable $\Delta n$ in an observed velocity variance spectrum. It is important to note that
271 Kolmogorov’s inertial subrange is a theoretical construct and therefore its wavenumber
272 (or frequency) range in a variance spectrum must be identified–hence the challenge to
273 investigators for eight decades. In fact, this explains the existence of various techniques
274 for detecting the subrange and why visual inspection remains acceptable. ARIIS is
275 essentially a series of systematic steps that are executed to determine the most probable
276 $\Delta n$ in a turbulence spectrum.
a. The Observed Turbulence Spectra

In order to develop ARIIS, the motion-corrected FLIP flux profile data was processed in 30-minute windows, with 50% overlap. Over this interval, records were rejected if the mean wind direction, $\theta_z$, was within the $\sim 150^\circ$ sector determined to be heavily influenced by flow distortion around the platform (see Ortiz-Suslow et al. (2019) for full analysis). This sector generally corresponds to winds with an easterly component, given $\text{FLIP}$’s mean heading of $290^\circ \pm 6.3^\circ$. The autovariance spectra were calculated for each 30-minute window using the detrended velocity record, a 4-term Blackman-Harris window, and MATLAB’s fast-fourier transform (FFT). Before starting the identification algorithm, the individual spectra were smoothed using a logarithmically-uniform bin average with 103 discrete bins (91 used for lowest two levels due to lower sampling frequency).

The observed autovariance wind spectrum can be normalized into natural, or surface layer, coordinates in order to account for the expected dependence on wind forcing and height into the turbulent, wall-bounded layer (Miyake et al. 1970; Kaimal et al. 1972).

Using the natural frequency, $f = nz/U$, the normalized 1-D variance spectra derived from equation 1 are:

\begin{align}
F_{uu} &= \frac{nS_{uu}}{u_*^2} = \alpha_u \left( \frac{\kappa z \varepsilon}{u_*^3} \right)^{2/3} \left( \frac{nz}{U} \right)^{m_{uu}} = \alpha_u \phi_e f^{m_{uu}}, \quad (11) \\
F_{\beta\beta} &= \frac{nS_{\beta\beta}}{\sigma_w^2} = \alpha_{\beta} \left( \frac{\kappa z \varepsilon}{u_*^3} \right)^{2/3} \left( \frac{nz}{U} \right)^{m_{\beta\beta}} = \alpha_{\beta} \phi_e f^{m_{\beta\beta}}, \quad (12)
\end{align}

where $\alpha_u, \alpha_{\beta}$ are component-wise, scaled versions of $\alpha$, $\sigma_w^2$ is the vertical velocity variance, and the dimensionless TKE dissipation rate, $\phi_e$, is used. At high $f$, $F_{uu}$ and
$F_{BB}$ should collapse onto a common slope over the inertial subrange (with slope $-2/3$, herein referred to as $m_0$) and separations at low $f$ should be controlled by $\zeta$. This was well-documented in the seminal surface layer study of Kaimal et al. (1972). Inspections of the CASPER-West data (not shown here) revealed a similar pattern as expected over low $f$, but with significantly more separation in the $\zeta$-segregated spectra over high $f$ than would be expected based on Kaimal et al.’s work (their Figures 3 and 4). During the general data inspections of CASPER-West, this unexpected behavior, which was observed to varying degrees across the entire profile, indicated potential for actual variability in the inertial subrange and motivated the development of ARIIS.

b. The Algorithm

The first step in ARIIS is to locate the inertial subrange bandwidth $\Delta f$. This was done automatically by relying on the first condition for Kolmogorov’s theory, namely that $\Delta f$ must occur over the isotropic portion of the spectrum. Equation 10 gives the isotropy coefficient, but programmatically it was simpler to define the isotropic ratio,

$$ R(n) = \frac{S_{uu}}{S_{ww}}, \tag{13} $$

without losing any generality. $S_{BB}$ have been smoothed, but not scaled using surface layer coordinates. Over the isotropic bandwidth, $R(n)$ converges on $3/4$ (Tennekes and Lumley 1972) and so a criteria-based test can be made to find the continuous range of $n$ over which $R(n) \approx 3/4$. It is noted that $R$ and $I$ are physically equivalent. The use of $S_{ww}$, instead of $S_{vv}$, in this ratio can be justified by the fact that $S_{ww}$ in the surface layer reveals an inertial subrange and collapses into a narrow band independent of at-
mospheric stability at higher frequencies than $S_{vv}$, especially under stable atmospheric conditions (Kaimal et al. 1972).

[ Figure 3 approximately here ]

This approach was applied by searching from low to high $n$ and finding the continuous subrange where $R(n) < R_c$ (i.e. the critical value for sufficiently isotropic), where the limits of $\Delta n$ had to satisfy this condition: $U/z < n < n_{Nyq}$, where $n_{Nyq}$ is the Nyquist sampling frequency. The lower limit of $n$ comes from the physical constraint on the turbulent eddies that can be resolved and that are not expected to be significantly distorted by the surface ($f \geq 1$). Typically, $f \geq 1$ is used as the onset of the inertial subrange, but ARIIS estimates the actual initiation frequency of the subrange, with this criteria as a lower limit. This makes ARIIS a unique and more general approach relative to other techniques, which either use visual inspection (e.g. Sjöblom and Smedman 2002) or arbitrary limits of variability (e.g. Yelland and Taylor 1996) of the spectrum without accounting for the $R(n)$.

For CASPER-West, if $U/z \leq 1/3 \text{s}^{-1}$, then the lower limit of $\Delta n$ was set to 0.333 Hz, to avoid surface gravity wave band contamination. Also, specific to CASPER-West, $R_c = 4/3$, which is a very generous cut-off that was determined by inspecting the output of ARIIS. If the final number of bins in $\Delta n$ was $< 4$, the spectra was rejected from the final data set (i.e. no $\Delta n$ found). As a further precaution, only an inner fraction of $n$-bins was passed on as $\Delta n$ (here 80% was chosen). The results of this process for an example spectrum are given in Figure 3.
ARIIS uses an iterative method to estimate the power law over $\Delta n$. This step is fundamentally different from previously published methods on detecting the inertial subrange because ARIIS does not assume that the isotropic bandwidth follows a $-5/3$ power law.

$F_{\beta\beta}$ can be statistically represented using a log-scaled linear relation:

$$\log(F_{\beta\beta}) = A + m_{\beta\beta} \times \log(f),$$

(14)

where $A$ and $m_{\beta\beta}$ can be determined via least-squares regression. Note that $m_{\beta\beta}$ is expected to be $m_0$, but will be considered a free parameter. The regression process enables calculating an independent measure of the uncertainty ($\mu$) for $m_{\beta\beta}$ based on the residual ($\delta$) between the observed $F_{\beta\beta}$ and the linear model. $\mu$ is a standard error estimate and is defined as: $\mu \equiv \sqrt{(\delta^2\Sigma_{i=1}^{N}f^2)/((N-2)N\sigma^2)}$, where $\delta^2/(N-2)$ is known as the mean squared error and $\sigma^2$ is the total variance in $F_{\beta\beta}$. $f^2$ is the sum of squares of $\log(f)$ and $N$ is the number of discrete bins in $\Delta n$.

A three-step robust fitting technique was used to determine $m_{\beta\beta}$: (1) log-scaled linear regression was executed over $\Delta f$; then (2), the Cook’s Distance, $D$, was determined for each fitted amplitude; and finally (3), any amplitude where $D > D_{\text{threshold}}$ were removed and the final regression was re-run using the filtered spectra. $D$ may be defined as an indication of the influence a single sample has on the regression. In this way, $D$ may be used to identify outliers in statistical regressions, without relying on dependent statistics, such as the mean, median, or variance. This metric was defined as:

$$D(i) = \frac{\Sigma_{j=1}^{N}(y_j - y_{j(i)})^2}{pMSE},$$

(15)
where $p$ is the number of predictor coefficients in the regression (here $p = 1$), $MSE$ is the mean squared error between the observed values and the original fitted response, $y_j$, and $y_{j(i)}$ is the fitted response excluding sample $i$. In other words, $D$ is calculated by iteratively fitting the observations, excluding observation $i$. Chatterjee and Hadi (1988) suggest a $D_{\text{threshold}} = 4/(N - p - 1)$ for samples with too much influence. The regression steps were completed independently for all $F_{\beta \beta}$, thus each component can exhibit a different power law over $\Delta f$. Spectra with $N < 3$ after the Cook’s Distance algorithm, were removed from the final data set (this was rare).

[ Figure 4 approximately here ]

It should be emphasized that any regression is an attempt to model a distribution of dependent data (here $F_{\beta \beta}$ v. $f$). Therefore, removing overly-influential samples is a necessary step in satisfying the prerequisites of the statistical modeling. In this sense, a sample flagged by ARIIS’ $D$ filter may not be an outlier in the layman’s sense (spurious or erroneous data that must be removed, see filtering technique described above), but this sample should be removed regardless because it exerts disproportionate influence on the regression model, as compared to the entire distribution.

An example of the fitting algorithm is given in Figure 4. The high $r^2$ in both cases, especially the improvement after applying $D_{\text{thresh}}$, demonstrates that ARIIS captures a log-linear portion of the spectrum. This is critical because this statistical form is only possible if $\varepsilon$ is independent of scale. Also, it is evident from this example that both $F_{uu}$ and $F_{ww}$ can be log-linear and exhibit different spectral slopes (i.e., $m_{uu} \neq m_{ww}$). The total impact of all the rejection criteria executed by ARIIS as applied to the
CASPER-West data set was an average 32.9 ± 0.011% data loss rate, across the entire profile. The overwhelming majority (96%) of spectra were rejected due to unfavorable wind directions. Therefore, making similar measurements not near a large platform or structure should improve the number of individual spectra that can be processed using this method.

[ Figure 5 approximately here ]

5. Example Results from CASPER-West

ARIIS was applied to 23 consecutive days of mast observations. As with any automatic processor, spurious or unphysical values may be included in the final data set. These values (e.g. $m_{\beta\beta} > 0$) were flagged and removed from the final processed data before analysis. For any given flux level, these outliers reflect at most a few tenths of a percent of the entire distribution. Here, $z$ refers to the global median height above the mean water level underneath FLIP’s boom.

a. The Inertial Subrange Bandwidth

The variability in $\Delta n$ identified by ARIIS is given in Figure 5. To the author’s knowledge, this is the first ever time record of the Kolmogorov inertial subrange bandwidth observed within the atmosphere. For the lowest measurement level, there was substantial variability in the low frequency limit of the inertial subrange. It was found that $\Delta n$ tended to decrease (become narrower) as $U$ increased, which was attributed to the constraint set by $U/z$ and the Nyquist frequency (10 Hz) of the lowest 2 anemome-
ters. When $U$ exceeded $10 \text{ m s}^{-1}$, it was possible that $\Delta n$ was outside of the resolved frequency range, i.e. no inertial subrange detected (gaps in Figure 5).

This degree of variability was less consistent for the upper measurement levels, which could resolve a larger range of turbulent eddy scales. In fact, for $z = 16$ m, over 60% of all $\Delta n > 10$ Hz, with the peak in the probability distribution being 0.42 at $\Delta n = 15$ Hz (see Figure 5). This indicates that the identified subrange spanned to the lowpass filter cut-off, therefore with a higher resolution sensor one would expect an even wider $\Delta n$.

[ Figure 6 approximately here ]

It is anticipated that some properties of $\Delta n$ may be related to thermal stability and wind speed. Figure 6 provides the low frequency limit (or initiation frequency, $n_i$) of the ARIIS-determined $\Delta n$ in both unstable ($\zeta < 0$) and stable ($\zeta > 0$) regimes, for four heights along the mast: $z = 3, 5, 12, \text{ and } 16$ m. These frequencies are limited at the lower range by $1/3$ Hz (used for CASPER-West to mitigate surface gravity wave influence) and thus the low frequency tails of these distributions collapse on this value. Figure 6 $n_i$ increases as $|\zeta| \rightarrow 0$ and as wind speed increases. This effect was pronounced closer to the surface, where $U/z \leq n_i$ creates a physical constraint on the range of isotropic eddies an ultrasonic anemometer can resolve (Figure 6dh). In the very near-neutral limit ($|\zeta| < 0.001$), the $n_i$ appears to become independent of $\zeta$, with wind speed being the only factor controlling the inertial subrange initiation (Figure 6bcg). In the cases of $z > 12$ m and $\zeta > 0$, $n_i$ exhibited little dependence on either $\zeta$ or wind speed (Figure 6ef).
While investigating the variability of $\Delta n$ and $n_i$ is interesting and novel, given the output of ARIIS, sonic anemometry is not best-suited for probing the frequency-dependence and characteristics of the inertial subrange. Therefore, this analysis was provided as primarily qualitative and the remainder this article will focus on the inertial subrange slope, which should be independent of the actual $\Delta n$—given a log-log linear spectrum.

[ Figure 7 approximately here ]

b. The Inertial Subrange Slope

Probability distributions, $\mathcal{P}$, of normalized $m_{uu}$ and $m_{ww}$ revealed considerable spread in the empirically-derived inertial subrange power law (Figure 7). $\mu$ was examined for both components and while there was some uncertainty in individual estimates of the slope, this was dominated by inter-sample variability. For both velocity components, $\mathcal{P}$ were fairly Gaussian, but with a height dependent mean and variance. In particular, for the highest flux level, the peak $\mathcal{P}(m_{\beta\beta}/m_0)$ was 1.05, which indicates a slightly steeper-than-expected slope over the inertial subrange. For the levels at or below 4 m, the peak $\mathcal{P}(m_{\beta\beta}/m_0)$ tended to be less than 1. This indicates a spectral slope that is substantially shallower than was predicted by Kolmogorov. Physically, this represents more energy per eddy scale over the inertial subrange. For $m_{uu}$, the peaks in $\mathcal{P}$ were 0.9 and 0.7 for $z$ at 3 and 4.04 m, respectively. However, the distributions for these levels were fairly broad even though there was only a 10% difference in the means. For the vertical-wind, the peak in $\mathcal{P}(m_{ww}/m_0)$ monotonically increased from 0.5 to just under 10.1175/JTECH-D-19-0028.1.
1.2 for $z$ from 3 to 15.8 m. For this component, there was over a 30% difference in the mean $m_{wW}$ between $z = 3$ and 4.04 m.

For all levels and velocity components, $P$ exhibited non-negligible spread (or uncertainty) in the actual value of the inertial subrange power law. This may be quantified using the relative error, defined here as:

$$e_r = \frac{\sigma}{\mu},$$

where $\sigma$ and $\mu$ are the sample standard deviation and mean, respectively. The levels at or below 4 m exhibited an $e_r \sim 23$-33%. This drops substantially for the upper flux levels, where $e_r$ ranged from approximately 12-17%. In general, variance for $m_{uu}$ was higher than $m_{wW}$.

[ Figure 8 approximately here ]

A time series of $m_{\beta\beta}/m_0$ revealed no distinct temporal or diurnal dependences (Figure 8). Most of the variance in the empirically-derived power law appears to be driven by mean environmental change, events, and random uncertainty. The lack of a diurnal dependence indicates that processes such as solar radiation and the diurnal wind (i.e. land/sea-breeze cycle), both of which had strong signals during CASPER-West, do not affect the inertial subrange. Interestingly, the $z$-dependence of $m_{\beta\beta}/m_0$ was not time-dependent and, from Figure 8, it is stronger for the $w$-component. In general, however, the variance in $m_{\beta\beta}/m_0$ was more dependent on the height above the surface, rather than the velocity component (i.e. $\beta = u$ or $w$).

While $m_{uu}$ did not exhibit a strong diurnal dependence, it did show some dependence on wind forcing and the mean azimuthal wind direction (Figure 9). In the following
description, \( r \) gives the correlation coefficient between the predictors (\( u_* \)) and response (\( m_{uu}/m_0 \)). Using \( z = 16 \) m as an example, an overall least-squares linear regression revealed a negative relationship between wind forcing (\( u_* \)) and \( m_{uu}/m_0 \) (\( r = -0.403 \)). However, for \( u_* < 0.2 \) there was significant scatter in the observations. When only considering westerly winds (240° < \( \theta_z \) < 300°), the overall variability decreases and the inverse dependence becomes more robust (\( r = -0.661 \)). The results for the northwest sector (300° < \( \theta_z \)) were interesting, where only a very slight negative dependence on forcing was observed, which was significant (\( r = -0.112, \ p = 0.028 \)). This trend strengthens (\( r = -0.352 \)) and steepens in the case of higher wind forcing, though this was not statistically significant (\( p = 0.12 \)) due to low sub-sample size (\( N = 21 \)).

[ Figure 9 approximately here ]

For easterly and southwesterly winds, the sample size drops to \( \sim 200 \) and no \( u_* \) values above 0.36 m/s were observed. Therefore, no significant trend with wind forcing was found. However, a two-sided student’s t test demonstrated that \( m_{uu}/m_0 \) for easterly and southwesterly winds were significantly different (\( p < 0.001 \)), with the former being on average 23.1% smaller than the latter. Before carrying out the test, both subsets of the data were normalized following Niaki and Abbasi (2007) in order to account for non-normal distributions. Due to the relatively high sample size for these statistical tests, the Cohen’s effect size (essentially a scale-normalized t statistic) was calculated to account for the impact of sample size. The Cohen’s value was found to be 1.38, which can be considered sufficient for a significant result. Therefore, regardless of the sample size, for the southwesterly and easterly conditions, the observed differences in \( m_{uu}/m_0 \) reflect
a physical change in the observed turbulence scaling as a function of the mean azimuthal wind direction.

6. Discussion

The algorithm developed here does rely on some heuristic or non-adaptive steps, like the setting of $R_c$. These non-adaptive features allowed for efficient processing of a large field data set, but they will always represent a limitation and trade-off for automatic processing. However, this highlights the fundamental issue of deciding what is ”good enough” for an approximation or convergence on a theoretical value. In the MASL literature there is no standardized approach for determining convergence and confidence bounds tend to be set by the investigator. For example, Yelland and Taylor (1996) arbitrarily used $\pm 30\%$ as a cut-off for a non-Kolmogorov power law, but there are examples with 10% or 20% deviations being used. These problems are not unique to this study, or even this field, but the analysis of these kinds of phenomena hinge on determining whether or not to consider a theoretical condition satisfied. The present work cannot definitively address this issue, but the results do motivate careful consideration and a standardized approach. The advantage of ARIIS, over other methods, is that it provides the information necessary for the investigator to make informed decisions about internal variability and quality of their data set.

[ Figure 10 approximately here ]

The CASPER-West data was used to develop ARIIS and provided an interesting opportunity for investigating the empirically-derived variability of the inertial subrange slope.
The results of this investigation revealed significant variance in the slope for both the along- and vertical-wind components. This variance could not be accounted for by extreme anisotropy (beyond the mean level observed), a non-log-linear spectrum over $\Delta f$, or other processing considerations. For example, the impact of varying the averaging window (Figure 10) or FFT windowing method (Figure 11) was evaluated. While on an individual basis, one expects differences in either the $\Delta f$ or $m_{\beta \beta}$ identified, when considered in aggregate, there was no statistically significant difference in the ARIIS outcome for these different processing steps.

ARIIS was used to analyze the dependence of $n_i$ on height above the ocean surface, wind speed, and stability, and Figure 6 may be the first ever empirical analysis of the inertial subrange initiation frequency in a geophysical turbulent flow. The most significant limitation to this analysis was the use of ultrasonic anemometers and the proximity to ocean surface. Given their limitations at high frequencies, it was not expected that the entire subrange would be resolved for every spectrum (Chamecki and Dias 2004). Furthermore, the proximity to the ocean surface coupled with the limits of anemometry sampling frequency, meant that there were limits to resolving $n_i$ when it became too low or too high. Theoretically, $\Delta n$, defined only by the $-5/3$ power law, is expected to widen from both low and high frequency limits with increasing flow speed (Tennekes and Lumley 1972). However, for a fixed height within the surface layer, as the wind speed increases the low frequency limit of the inertial subrange (i.e. $f = U/z$) moves to higher and higher frequencies, constraining $\Delta n$ to a much narrower range than would
be predicted by $-5/3$ alone. This demonstrates the need to account for isotropy when
determined the location of the inertial subrange.

In absolute terms, the variance in $m_{ββ}$ tended to be constrained within ±20% of the
mean, which itself was approximately within 10% of $m_0$. The major exceptions were
for the two lowest observing levels of the FLIP mast. Therefore, as an example, the
majority of these observations would have passed the Yelland and Taylor (1996) criteria. However, more detailed investigations revealed that this variance was non-randomly
distributed and revealed systematic dependence on wind forcing and direction. In the
case of west to northwesterly winds, increased wind forcing was associated with slopes
shallower than Kolmogorov’s expected value. Also, for the same wind forcing, easterly
winds exhibited significantly shallower slopes than southwesterly winds. In this region,
changing wind direction will most likely affect the interaction between the turbulent
flow and the underlying wave field. The intensity of the wind-generated waves corre-
lates significantly with friction velocity (e.g. Kraus and Businger 1994), and therefore
the distribution in Figure 9 is a preliminary indication of a relationship between the in-
ertial subrange slope and wave activity. This interaction is somewhat unexpected, given
the frequency range of the inertial subrange, and more work is needed to fully under-
stand this mechanism. While there may be some impact from changing fetch, given
the coastline and islands, this work was solely focused on the high frequency range of
the wind variance spectrum, which would be expected to adjust to a changing surface
roughness rather quickly. The closest land area was Santa Barbara Island, which was
nearly 45 km from FLIP and has an area of ~2 km². The results of the analysis into
the CASPER-West observations revealed significant and unexpected variability in the empirically-derived inertial subrange slope, however a full exploration of the underlying variability in the slope goes beyond the scope of the present work. This effort is the focus of continued study, some of which is presently in preparation.

[ Figure 11 approximately here ]

7. Summary

Presented here was the design and implementation of a novel approach to detecting Kolmogorov’s inertial subrange in velocity variance spectra. The novelty of ARIIS is not in the exercise, since ad hoc inertial subrange detection methods have been utilized since its 1941 proposal. Rather, ARIIS lays out a series of controlled steps for analyzing a single velocity variance spectrum and identifying the most probable location of the subrange. This is executed adaptively and efficiently, which is useful for large data sets, where visual inspection imposes an onerous burden on the investigator. While ARIIS is fundamentally based on Kolmogorov’s original considerations (i.e. isotropy), the design of the program makes a concerted effort to minimize reliance on theoretical assumptions about turbulence variability. Specifically, the determination of the actual onset of the isotropic subrange and a robust measurement of the observed inertial subrange slope makes this approach novel as compared to other methods in the study of turbulence within the ASL—and in general, for turbulence cascade studies in both atmospheric and oceanic domains.
ARIIS was used to investigate the empirically-derived slope in turbulence measurements made from FLIP. As a result of ARIIS’s novel approach, this study provided the first analysis of the complex variability of the observed inertial subrange bandwidth and initiation frequency within the atmosphere. In addition, the findings revealed significant variability in the observed inertial subrange slope that could be attributed to both random variance as well as a systematic dependence on the mean forcing within the MASL. In particular, the slope exhibited a dependence on the mean wind forcing and the mean azimuthal wind direction. This suggests that the turbulent energy distribution within the airflow above the ocean surface is dependent on the mechanical interaction between the wind and waves and that this interaction changes with the relative velocities of these two fields. Exploring this mechanism and its implications is currently the focus of further investigation on this particular data set, but should also be the aim of studies over the general ocean to better understand the prevalence of this process.

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tion portion of this work. Richard Lind (NPS) is appreciated for his help in preparation for CASPER-West. The data set used in this analysis is available here: https://nps.box.com/s/dyfswo8ly0qwmgsi3nmfyo06265nk0kz; and the ARIIS code is available at: https://github.com/dortizsu/ARIIS.

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Ortiz-Suslow, D. G., J. Kalogiros, R. Yamaguchi, D. Alappattu, K. Franklin, B. Wauer, and Q. Wang, 2019: The Data Processing and Quality Control of the Marine Atmo-


LIST OF FIGURES

Fig. 1. Location of CASPER-West field campaign with location of FLIP marked. The inset shows the port boom and meteorological mast. FLIP’s mean heading was ≈290° from True North. .......................... 40

Fig. 2. Times series of 30-minute mean wind speed, \( U_z \) (top), and direction, \( \theta_z \) (middle), as well as atmospheric stability, \( \zeta \), over the CASPER-West study period. Only the data that were passed onto ARIIS are shown. \( U_z \) was not corrected for non-neutral stability and represents the mean at height \( z \). ........................................... 41

Fig. 3. Example of ARIIS-identified inertial subrange. (a) The vertical-dashed lines mark the low frequency limit and high frequency limit (\( n_c \) could be \( n_{Nyq} \)) from ARIIS. The red-dashed lines mark \( R_c (4/3) \) and the expected isotropic value (3/4). (b) The velocity variance spectra for this example, with Kolmogorov’s \(-5/3\) slope (black-yellow). The shaded green bandwidth is the inner 80% passed on as the inertial subrange \( \Delta n \). This sample comes from \( z = 8.46 \) m at 01:30 September 30 UTC. The mean \( U_z \) and \( \theta_z \) was 6.04 ms\(^{-1}\) and 278.2°, respectively. .......................... 42

Fig. 4. Example of robust fitting technique applied by ARIIS to the \( u \) (left) and \( w \) (right) wind components, using the example spectra from Figure 3. The solid black line has a log-log slope \(-5/3\). ........................................... 43

Fig. 5. Time series of the final bandwidth, \( \Delta n \), outputted from ARIIS for the lowest (blue) and upper-most (red) flux levels. Inset shows probability distributions, \( P \), of \( \Delta n \), the number of bins in histograms was held constant. ........................................... 44

Fig. 6. The ARIIS-detected inertial subrange initiation frequency (\( n_i \)) as a function of \( \zeta \) for unstable (a-d) and stable (e-h) conditions, with the points colored by \( U_z \) up to 5 ms\(^{-1}\). The high wind event (10/20 00:00 - 10/21 12:00 UTC) is marked in x’s. In each panel, the horizontal line marks the \( n_i = U/z \) limit for \( U_z = 5 \) (in a and e, this falls below the axis limits); vertical checkered lines denote \( |\zeta| = 0.1 \). ........................................... 45

Fig. 7. Probability distributions, \( P \), of \( m_{uu}/m_0 \) (left) and \( m_{ww}/m_0 \) (right) for four measurement levels of the profile. Unity indicates agreement with Kolmogorov’s theory. ........................................... 46

Fig. 8. Time (left) and hourly (right) dependence of \( m_{uu}/m_0 \) (top) and \( m_{ww}/m_0 \) (bottom). The hourly dependence is given as a box-and-whisker: the mean (centroid of box/horizontal line), median (dot), and the 50% and 95% intervals (box edges and error bars, respectively). Due to graphing limitations, some whiskers (i.e. error bars) are occluded by their boxes. ........................................... 47

Fig. 9. Normalized \( m_{uu} \) as a function of wind forcing (\( u_* \)) segregated into different wind direction sectors, \( \theta_z \), for the observations from the upper-most flux level. The number of individual samples per sector is given in the figure key. Linear regression lines for all observed (black-white checkered), westerly (blue), and northwesterly (gold) wind directions are shown. The red line marks \( m_{uu} = m_0 \). ........................................... 48
Fig. 10. $\mathcal{P}$ of $m_\beta/m_0$ using four different window lengths. Note: these $\mathcal{P}$ include some spurious output from ARIIS (less than 1% of total distribution)—these values were filtered out of the dataset analyzed in Figures 7-9.

Fig. 11. Same as previous, but comparing different windowing techniques. The Blackman-Harris window was used to generate the results presented here (Figures 7-10), but the Hamming window was recommended by Kaimal and Kristensen (1991). Both windowing techniques were applied over the entire domain and applied using MATLAB’s native functions in default mode (see MATLAB documentation).
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FIG. 4: Example of robust fitting technique applied by ARIIS to the $u$ (left) and $w$ (right) wind components, using the example spectra from Figure 3. The solid black line has a log-log slope $-5/3$. 

$\nu$ (left) and $\omega$ (right)
Fig. 5: Time series of the final bandwidth, $\Delta n$, outputted from ARIIS for the lowest (blue) and upper-most (red) flux levels. Inset shows probability distributions, $P$, of $\Delta n$, the number of bins in histograms was held constant.
Fig. 6: The ARIIS-detected inertial subrange initiation frequency ($n_i$) as a function of $\zeta$ for unstable (a-d) and stable (e-h) conditions, with the points colored by $U_z$ up to 5 ms$^{-1}$. The high wind event (10/20 00:00 - 10/21 12:00 UTC) is marked in x’s. In each panel, the horizontal line marks the $n_i = U/z$ limit for $U_z = 5$ (in a and e, this falls below the axis limits); vertical checkered lines denote $|\zeta| = 0.1$. 

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FIG. 8: Time (left) and hourly (right) dependence of $m_{uu}/m_0$ (top) and $m_{ww}/m_0$ (bottom). The hourly dependence is given as a box-and-whisker: the mean (centroid of box/horizontal line), median (dot), and the 50% and 95% intervals (box edges and error bars, respectively). Due to graphing limitations, some whiskers (i.e. error bars) are occluded by their boxes.
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$48$
Fig. 10: $P$ of $m_{\beta\beta}/m_0$ using four different window lengths. Note: these $P$ include some spurious output from ARIIS (less than 1% of total distribution)—these values were filtered out of the dataset analyzed in Figures 7-9.
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